14-19 Mathematics and ICT: Curriculum and Assessment Issues

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1 Introduction

Background
The QCA commissioned work on developing 14-19 Curriculum Pathways: Phase 1 in January 2005. This went to two consortia: Kings’ College/Edexcel and Leeds University. The final reports of these consortia are now (August, 2006) with the QCA. In June 2006 QCA awarded a 14-19 Curriculum Pathways: Phase 2 contract to the AQA/Leeds University consortium and other Phase 2 contracts are expected. These will pilot and trial developments recommended in Phase 1 reports. These recommendations include recommendation regarding the use of ICT in mathematics.

At the same time as Phase 1 work was being carried out, QCA commissioned work to identify a set of principles about qualification design and criteria that will enable ICT to be used more effectively in mathematics. Appendix 1 presents the draft brief and explains the goals of Strands 1 – 4. Strand 1 resulted in three sets of ICT-mathematics vignettes from researchers at the universities of Bristol and Cambridge and at the Institute of Education. Strand 2 was addressed by a team of eight people who discussed matters and wrote papers and sample assessment items for mathematics using ICT. Aspects of work developed in Strands 1 & 2 will be reviewed in this report.

The brief for this report was to address “the region between strand 2 and strand 3, possibly outlining or touching on issues in strands 3 and 4” and to develop principles and practical ideas that could feed into Phase 2 work (but that this should not be geared to any one Phase 2 contract).
The scope of this report (areas addressed and not addressed)
This report is not a review of research. Indeed, references in this report are selective and have been kept to a bare minimum.

Pedagogical matters are crucial to a future where ICT use is integrated into the teaching and learning of mathematics but pedagogical matters are not within the brief given to the author. If aspects of this report are taken further, then it will be important to liaise with the National Centre for Excellence in Teaching Mathematics (NCETM) on pedagogy and continued professional development (CPD) for teachers of mathematics.

This report addresses matters concerned with the integration of ICT into the mathematics curriculum and its assessment and is not concerned with e-assessment *per se*, though matters relating to e-assessment will arise in this report, these will relate to other substantive issues.

Sections 2-6 primarily concern GCSE and GCE curriculum and assessment issues. Section 7 considers other course and qualifications and sections beyond this are generic.

‘Assessment’ in this report concerns summative assessment (examinations), not formative assessment. There are many ways that ICT can contribute to formative assessment but it is simply not in the brief of this report to consider this matter.

Examinations considered will be mathematics examinations: existing examinations, e.g. GCSE, and proposed examinations, e.g. functional mathematics. The brief of this report is to consider the integration of ICT into these examinations and not to propose further qualifications such as *Mathematics with ICT*.

The structure of this report
The next section raises issues addressed in the *Strand 2* working group and briefly opens up a host of matters, many of which are considered in more detail in later sections. Graphic calculators are then considered. Section 4, on target groups, raises issues concerned with different categories of students. The next two sections consider curriculum and assessment matters respectively. The discussion up to this point is largely, but not exclusively, concerned with current mainstream courses and qualifications, especially GCSE and GCE. Section 7 specifically raises issues concerned with other courses and qualifications, existing and proposed. Section 8 sets out possible curriculum and assessment principles. The final section summarises issues, makes tentative recommendations for the use of ICT in specific qualifications and outlines possible trialling and piloting for the immediate future.
2 Strand 2

Strand 2 was addressed by a team of eight people who met for three full day meetings between February and May, 2006. They were:

Nick Doran Qualifications and Curriculum Authority (Chair)
Sheila Messer International Baccalaureate Organization (IBO)
John Monaghan University of Leeds, 14-19 Pathways Phase 1
Roger Porkess MEI
Ian Stevenson King’s College London, 14-19 Pathways Phase 1
Ron Taylor Hampshire LEA
Geoff Wake University of Manchester, AS Use of Mathematics
Einir Wyn Davies International Baccalaureate Organisation

Their brief is that given in Appendix 1 but no consensus was reached. A separate report on the work of this group is forthcoming from QCA, so this section merely raises issues addressed by this group. Raising these issues is useful for the purposes of this report as it serves to illustrate different perspectives – differences which are likely to be reflected in the mathematics education community in the UK. The two representatives from the IBO reported on the integration of graphic calculators into IBO mathematics curriculum and assessment and this is the focus of the next section of this report.

The group discussed whether the recommendations should be made with regard to the use of specific hardware and software. Value was seen in a wide range of software and microworlds but three generic software systems were seen as particularly relevant: spreadsheets, dynamic geometry and function graphing packages. Computer algebra systems and statistical packages were viewed positively but: it was recognised that computer algebra systems were viewed with great suspicion by a significant body of people who were concerned that they might undermine students’ algebraic skills; the current debate on the place of data handling in GCSE Mathematics, and the expectation that data handling in the curriculum will be reduced in scope, appeared to discourage concrete recommendations for the use of statistical packages.

There was disagreement on the place of spreadsheets in the curriculum. Although all recognised their value (for student work and for skills beyond the classroom) some thought their use should be privileged beyond other software. It was recognised that ‘spreadsheet mathematics’ was not the same as pencil-and-paper mathematics. This, of course, applies to all software as tool use transforms mathematical actions; the debate centred on the wisdom of privileging one alternative form of mathematics above other forms.

Hardware offers opportunities but these opportunities are limited with regard to numbers of machines, cost and range of mathematical application:

♦ Schools do not currently have sufficient desktop computers to ensure that computers can be integrated into compulsory examinations and this situation is likely to persist for some time.
♦ Laptop computers offer a way to alleviate the desktop problem and offer portability but a concern is with costs.
♦ Graphic calculators can overcome the above problems but their functionality, compared to computers, are currently limited.
♦ Issues of pre-loaded programs giving some students an unfair advantage present additional problems with all hardware options but are less acute with graphic calculators.
The group discussed the question ‘In what way is the ICT integral to, rather than being a vehicle for, the assessment?’ There was wide disagreement on this question despite the fact that all agreed that the current ethos of ‘teaching for the test’ meant that curriculum changes should be matched with changes in assessment. Advantages to ICT being a vehicle for assessment include having assessment ‘on demand’, allowing students to demonstrate competence when ready but this may not be possible for mature students who may have only eight months of study. The group included members who represented ‘pole positions’: one advocating 100% on-screen assessment with no recourse to other media and one who advocated that students should use whatever tools they wished including wholly paper-based media for questions expecting the use of technology. A related but distinct disagreement concerned the assessment of ICT skills on mathematics examinations and again ‘pole positions’ were present in the group: those who saw certain ICT skills as mathematical skills and one who defended the thesis that “we use technology but we assess mathematics”. Another issue concerned marks and grades. It was recognised that assessment, especially at GCSE and GCE, has evolved so that questions can provide fine grade distinctions but that assessment integrating ICT would require time for fine grade distinction marking to be equitable. Two members of the group suggested that an interim arrangement, where the ICT component would receive a ‘pass-or-fail’ award which would validate or not the main qualification, should be considered.

The final remit of the group was to “develop tasks with assessments that would require the use of ICT”. There appeared to be some unanimity amongst group members but this was unanimity regarding the problem of the form of the examination. Coursework was viewed as a suitable means to assess the use of ICT in mathematics but current teacher dissatisfaction with coursework and problems concerned with plagiarism led the group to refrain from recommending coursework as a means to assess the use of ICT in mathematics. Assessing the use of ICT in mathematics via traditional examinations was seen as extremely problematic for platforms other than graphic calculators due to reasons of ensuring that schools have sufficient suitable computers and of ensuring that computers do not have pre-loaded information which may benefit students in unwanted ways.

The possibility of using prior data sheets, such as are used in Free-standing Mathematical Qualifications (FSMQ), was discussed. Prior data sheets in FSMQ are issued to students several weeks before the examination and new sheets are given to students in the examination. They give students the opportunity to prepare for the examination. In doing this they go some way to providing a coursework type of experience, in that they may explore a problem area or context, without fear of plagiarism. Being linked to a timed examination, however, they do not, however, overcome the problems noted above of schools having sufficient suitable computers and of ensuring that computers do not have pre-loaded information. The group felt that the use of prior data sheets in the assessment using ICT was something worthy of further consideration.

Appendix 3 presents two spreadsheet and three dynamic geometry assessment tasks produced by group members (these are the tasks available electronically). The following comments on the tasks. The two spreadsheet tasks, which were written by different authors, are both presented in a prior data sheet format. The group endorsed both tasks as potentially suitable for assessment but the second task was viewed as too ‘leading’, e.g. directing students to go “from 1 to 20 in steps of 1 gallon”. The three dynamic geometry tasks were written by the same author and the form of the examination is left open.
3 Graphic calculators

Graphic calculators (GC) have been available for 20 years but they now commonly include a range of computing tools: Cartesian, parametric and polar graphing capabilities; scientific calculator with built-in numeric routines and functions; programming language; tabular functionalities akin to a spreadsheet; and sophisticated data-handling functionalities.

The English experience of using graphic calculators is at odds with much of the developed world: in the early 1990s there were virtually no restrictions on their use in examinations (when there were restrictions in many other countries) but by the late 1990s restrictions were imposed (and relaxed in many other countries). These restrictions appear to have led to a reduction in their use in mathematics classes in the last 10 years in England, e.g. a teacher reported in Rodd & Monaghan (2002), “They’ve effectively been written out of the curriculum, hence they will not be used” (ibid., p.100). Rodd & Monaghan also report teachers saying that “pressure is to use computers rather than graphic calculators” (ibid., p.99). The English GC experience to date is thus probably not the best one to focus on with regard to how they be used to integrate technology into future curricula and assessment.

The International Baccalaureate Organisation (IBO), however, has adopted a progressive and considered approach to the use of GCs (which they call graphic display calculators – GDCs) into curriculum and assessment. For this reason Brown & Davies (2002), which reports on the introduction of GDCs into assessment, is summarised below.

The use of GDCs in examinations was approved in 1992 for use in examinations in 1995. It was intended that students could use any calculator and that questions would be calculator neutral. In 1997 the regulation was changed so that from 2000 onward ‘calculator neutral’ became ‘calculator assumed’. In 1997 it was agreed that questions should be set at a conceptual level and not at a calculational level. The intention was to set questions to test the students’ understanding as opposed to their ability to use algorithms to find solutions.

The first discussions were about how questions needed to change to meet the graphic calculator requirement. By 1998 issues related to mark schemes and appropriate responses from students were discussed. There was also a change of mind regarding working done on a calculator without any written evidence so that “if a numerical answer could be obtained directly from a calculator then, in examinations, questions should direct the candidate to 'write down' the answer” but this clashed with established marking practice where a correct answer with no indication of method normally received no marks.

Brown & Davies state that “setting of questions at a more conceptual level has been more difficult than was expected ...the difficulty has been caused in part by the examiners’ limited experience in the setting of conceptual questions.” Another problem was making advice agreed between IBO officers and Chief Examiners clear to school teachers.

Brown & Davies then consider developing examination questions and note that examiners often start from a standard question and modify it for GDC use. They consider one question (Mathematical Methods, Standard Level, 1998 Specimen Paper).
In this question you should note that radians are used throughout.

(a) Use your graphic display calculator to show the graph of \( y = \pi + x \cos x \) for \(-0.5 \leq x \leq 0.5\).

Sketch the graph on squared paper, using a scale of 2 cm per unit, making clear:
(i) the scale on each axis;
(ii) the approximate positions of the intercepts and the turning points.

(b) Show that \( \pi \) is a solution of the equation \( \pi + x \cos x = 0 \).

(c) Find another solution of the equation \( \pi + x \cos x = 0 \) for \(-0.5 \leq x \leq 5\) giving your answer to six significant figures.

(d) Let \( R \) be the region enclosed by the graph and the axes for \( 0 \leq x \leq \pi \). Shade \( R \) on your diagram.

(e) Write down an integral, which represents the area of \( R \), and use your calculator to evaluate this integral to an accuracy of six significant figures. Show that your result corresponds to \( \pi^2 - 2 \).

Brown & Davies consider the comments of examiners and advisors with regard to the wording of the question, method of solution and GDC capabilities. With regard to the wording of a question they note that examiners often specify a particular requirement to ensure that a new innovation is fully incorporated into the assessment. But, as one examiner commented, “Do they need to be told to use a GDC? I suppose it is pretty obvious, given the six significant figures.” A further issue of concern is the expectation for the solution to part (c). Is it to use the zoom and plot facilities of the calculator or is it to use the built in functions of the calculator to find these points?

With regard to the method of solution a difficulty when writing questions is our traditional expectations: when a student is asked to show that a result is true, we expect them to provide an algebraic approach but technology presents alternative methods of solution, which can present unexpected difficulties for examiners and question setters. What are markers to make of, say, "I put 3.1415926 in the equation and it worked", or "On the GDC there is a root at 3.14159"? Should the wording be changed from ‘show’ to ‘verify’ or, perhaps, ‘Use trigonometry to verify that....’ with the rider that they show all steps? Conflicts arise in examiners’ minds around the use of traditional and numeric methods and what counts as an acceptable proof.

With regard to the capabilities of graphic display calculators a problem is the variety of GDCs (different brands and different models of a particular brand). Examiners need to consider the expectation of questions and the functionality of the calculator. In reference to part (e) of the question an examiner commented that unless a minimum specification was set, then schools would complain and that under-specification should be avoided. A further issue is that students are likely to incorporate machine-specific language and notation into their answers.

Brown & Davies go on to consider the functionalities and syntax of different calculators. With regard to functionalities they consider significant differences statistical facilities available over GDCs with regard to a question on the normal distribution and Chi-square test. Apart from equity issues concerned with one calculator doing more than another there are issues regarding awarding marks.

Further to this unexpected calculator-specific results can occur “and can be a trap for the unwary”. They consider a series of questions related to the function \( y = x(x^2 - 4)^{2/3} \) which was written as an interesting function ideally suited to the calculator. On one calculator the syntax changed the graphic display considerably:
Brown & Davies ask how examiners to deal with this and similar problems (the paper describes how the correct determination of a point of inflection of this function using a GDC depends on the starting value).

The remainder of the paper considers teachers’ comments on questions and examiners’ responses. Brown & Davies conclude:

♦ that even with considerable time for preparation, and many workshops offered throughout all regions, there has still been a slower uptake on the use of technology than was anticipated,

♦ even with 3 years lead in time (and specimen examination papers available 2 years prior to the first examination) some schools were still not fully prepared,

♦ although the examiners grew in confidence and skill in the use of a graphic calculator ... it was evident that students either did not know or were unwilling to use these functions,

♦ we do not believe questions should be written which explicitly state that a calculator will be required. Questions should provide the opportunity for students to choose the most appropriate mathematical tool at their disposal. ... it is important that students are given the opportunities to experience tool selection,

♦ it is essential to determine the expectations of that technology when it comes to assessment. For the IBO, it was deemed necessary to set minimum specifications for calculators due to the diversity of available models,

♦ writing questions for examinations where the use of graphic calculator is required is a skill that needs to be learnt, therefore training needs to be provided for examiners in the same way as for teachers.
4 Target groups

14-19 mathematics includes 14-19 year old students at Entry level and levels 1, 2 and 3 as well as returning-to-study adults. This is a wide range of students. This report differentiates between levels rather than ages though there are obviously issues, not considered here, concerned with differences in the appropriate mathematical ICT experiences for teenagers and for adults and of the short period of study time for many returning-to-study adults.

This report does not address Entry level. This small but important and often disadvantaged group is extremely diverse, ranging from school students with diverse learning problems to immigrants, often refugees, with cultural backgrounds where ICT often has little or no place. An important factor in working with Entry level students is setting work at a level where they can succeed and develop cognitive skills and confidence in the mathematics they do. Given this cognitive and cultural diversity and also the importance of not undermining these students’ confidence, no specific recommendations are made for the place of ICT in their mathematical curriculum and assessment. This is not to say that there is no place for ICT in their curriculum and assessment but, rather, that it is the place of the teacher to judge whether the use of ICT will increase these students’ engagement with mathematics or not.

With regard to levels 1, 2 and 3 mathematics, a division is often made to consider levels 1 and 2 together and level 3 separately. This makes sense from a school age point of view as levels 1 and 2 are the domain of KS4 and level 3 mathematics is post-compulsory mathematics (though many students do study levels 1 and 2 mathematics beyond compulsory schooling). From a mathematical point of view, however, there is a case that levels 2 and 3 go together better and that level 1 is the one on its own. The reason for this is that mathematics at levels 2 and 3 mathematics is more abstract and algebraic than at level 1, which contains very little algebra. This has important implications for the integration of ICT into mathematics curriculum and assessment that can be viewed from a software perspective and a from student perspective.

From a software perspective algebra is a powerful tool which allows complex relationships to be expressed in spreadsheets, graphic packages, computer algebra systems and statistical packages. An ICT focus which celebrated students working with these complex relationships on these packages would be unlikely to generate rewarding techno-mathematical experiences for level 1 students. From a student perspective an initial focus on level 2 or level 3 would be likely to focus again on how algebraic ideas may be used with a possible result of curriculum developers scrambling around to find techno-mathematical activities suitable for level 1 students.

The upshot of these considerations is a need to consider, and perhaps even prioritise, ways in which ICT can be integrated into the teaching, learning and assessment of level 1 mathematics. This would place the most difficult challenge at the forefront and may avoid a situation in which level 1 students’ techno-mathematical experience is impoverished compared to level 2 and 3 students’ experiences.
5 Curriculum matters

There are many ways of viewing the mathematics curriculum, from a formal document listing content and processes to the activities that students experience in mathematics classrooms. Further to this the mathematics curriculum should be considered with regard to the age of the students for the curriculum for early years children should differ from the curriculum for young adults. This report concerns adolescents and young adults. It begins with a consideration of curriculum documentation and leads to activities that students may experience in their mathematics education.

There are two traditional ways to specify curriculum documentation, a content curriculum and a process curriculum. The current KS4 National Curriculum is an example of a primarily content curriculum and the draft (of 31/7/06) KS4 Programmes of Study is an example of a primarily process curriculum.

There are real problems with integrating ICT into a primarily content-based mathematics curriculum because ICT transforms mathematical activity, i.e. a graph plotter may draw a graph similar to a hand sketched graph but what is done in each instance is very different: by hand students are will calculate, tabulate, scale axes and plot points; with ICT students will input functions and zoom-in. It would be extremely difficult to reconcile, say, the NC levels of these different actions. Going beyond this specific example and considering all specific examples of possible ICT use in content areas, the task of intelligently assigning ‘with-ICT levels’ and linking these to ‘by-hand levels’ appears ill-founded.

The integration of ICT into the mathematics curriculum appears to be less problematic with a process-based curriculum as the focus is on actions. The draft (of 31/7/06) KS4 Programmes of Study presents four key processes: representing, analysing, interpreting and appreciating. There appears to be scope to integrate ICT into such a curriculum. Indeed, the first two sets of Strand 1 vignettes, see Appendix 2, partially suggest a process curriculum approach where the processes are: conjecturing, proving and representing. Before taking this idea further it is useful to consider the mathematics curriculum as activities that students experience in mathematics classrooms.

Both of the above approaches (content and process) are somewhat at odds to an approach to the curriculum which emphasises the activities that students experience in mathematics classrooms. This approach could crudely be described ‘students constructing mathematical meaning in resource-mediated activity’. This approach is not primarily concerned with ensuring that students cover predetermined content and/or process; the emphasis is on mathematically worthwhile activity. This approach is evident in all three Strand 1 vignettes but is especially evident in the London Knowledge Lab vignettes. It is likely that some QCA staff were disappointed by this set of vignettes. If this is the case, then it is not so much a matter of any group being ‘right or wrong’ but the curricula foci that different working practices, e.g. QCA and research, generate. Different approaches to the curriculum are likely to generate tensions between people working with different approaches and it is likely that these tensions will arise.

1 References to ‘approaches’ in this section are stereotypes. In reality people hold hybrid approaches.
2 This does not come across in the description of this set of vignettes in Appendix 2 simply because there is not space in this report to describe details of student work in the microworlds presented.
Similar tensions are likely to exist with 14-19 mathematics teachers. Of the myriad of classroom curricula approaches to the use of ICT, including avoiding anything other than a scientific calculator, there are teachers who like to have clear content curriculum reasons for using specific ICT packages, e.g. using Omnigraph to illustrate specific geometric transformations, and teachers who see ICT as a resource for students to explore ideas, e.g. using Omnigraph to allow students to explore different ways of enacting transformations. What these teachers have in common are routines which use ICT to enact bits of their classroom curricula; very few teachers attempt a systematic integration of ICT into anything more than ‘curriculum bits’.

Returning to whether the curriculum focus of the integration of ICT into the mathematics curriculum should be on content or on process, a focus on process appears more likely to be successful in terms of the classroom curriculum: the above stereotyped ‘ICT as a resource for student exploration of ideas’ teacher would be unlikely to find this problematic; the above stereotyped ‘ICT to illustrate concepts’ teacher would have to involve more than ‘appreciating’ but their established routines would not have to be jettisoned.

Another reason why the integration of ICT into a process curriculum may be more desirable is that the summative assessment of a process curriculum does not need to try to cover a somehow ‘representative-of-mathematics’ content (an immense and, very likely, a futile undertaking) as mathematical processes span content.

The four key processes of the draft KS4 Programmes of Study (representing, analysing, interpreting and appreciating) appear to be suitable processes on which to base the integration of ICT into the curriculum. The comments on the suitability of the draft KS4 Programmes of Study apply to the integration of ICT into the curriculum. It is likely that teachers, many of whom acknowledge that they ‘teach to the test’, will find it difficult to adapt their routines to such a curriculum but the use of ICT just could assist the development of adapted routines.

It would be useful in future drafts of the KS4 Programmes of Study to question whether the many references to ICT are indeed useful. Their presence could well encourage teachers to think “this is where you use ICT” which could lead to an impoverished enacted curriculum and stifle new ideas.

Statistics in the curriculum
The current review of the place of statistics in GCSE Mathematics, and ramifications for statistics in other courses, presents problems for considerations of ICT and statistics – it is not possible at the time of writing to consider details. Some general comments, however, can be made.

Statistics and ICT go together extremely well. Statistics is an area of professional mathematics that has undergone significant development due to the influence of ICT. Statistics with ICT is an area of school mathematics actually used outside of the classroom. ICT is particularly suitable to provide students with opportunities for activities which illustrate some central concepts of statistics in a way that was previously not possible, e.g. tasks designed around repeated experiments to help students understand the central limit theory. A great deal of statistics can be done on a graphic calculator. Indeed, GCs are possibly more important to statistics at GCE level than graphing is to GCE as a whole.
The integration of ICT into a process, as opposed to a content, curriculum appears to alleviate potential problems in restrictions of what can be said about ICT and statistics at the time of the review in as much as a process approach allows a ‘freeze’ on considerations of statistics with the possibility of addressing ICT and statistics thoroughly once the place of statistics is clear.
6  Assessment matters

The integration of ICT into the assessment of mathematics is likely to be a controversial matter if the ‘calculator debate’ is anything to go on. This debate, which concerns the use of the calculator in learning and teaching, not just in the assessment of mathematics, has raged for over 25 years and has, perhaps, divided the wider mathematics education community more than any other issue:

This article proposes that paper-and-pencil arithmetic no longer be taught in elementary school and that it be replaced by a curriculum which emphasizes mental arithmetic much more than at present and in which calculators are used for instructional purposes in all grades including kindergarten. (Ralston, 1999, p.1)

Many good students, when challenged to simplify \((\sqrt{3})^2\), now have no response available to them other than to reach for a calculator ... the superficial observer may mistake this for mathematical behaviour. It is not! The calculator here is no longer an ‘aid’. Instead it controls, obscures, and distorts the meaning of the symbols and of the operations. (Gardiner, 1995, p.529)

In England there are many organisations representing different sections of the mathematical community and most of these organisations adopt a position on the place of calculators in the mathematics curriculum and its assessment (though generally expressed in a more tempered manner than the ‘pole positions’ expressed in the quotes above). The establishment of the Advisory Committee on Mathematics Education (ACME) in 2002 was intended as a means for the mathematical community to act as a single voice. A 2005 publication (ACME, 2005) focused on assessment in 14-19 mathematics and arguably represents a voice of compromise in the debate.

ACME (2005) recognises that the appropriate use of computers and calculators can enhance the teaching and learning of mathematics and that “assessment mechanisms should recognise and reward the proper use and mastery of computer software ... [but] assessment can be a problematic and contentious issue”. They consider that “assessment of the use of spreadsheets and other software packages could be envisaged for appropriate work” and that computers could provide a means for ‘assessment-when-ready’ but there is a danger that such assessment will focus on short structured questions at the expense of higher level skills. They recommend that “More research is needed on the use of computer-based assessment in 14–19 mathematics”.

ACME (2005) could be criticised for saying very little on and/or ‘sitting on the fence’ with regard to the integration of ICT into assessment but it serves to highlight this as a “contentious issue”. ACME (2005) also introduces issues around e-assessment. Although the brief of this report (14-19 Mathematics and ICT) is not concerned with e-assessment per se, e-assessment matters cannot be completely ignored. Threlfall & Pool (undated) report on QCA sponsored work on using computers in the assessment of mathematics, specifically “what may be lost and gained by undertaking mathematics assessment on computer” (ibid., p.1). Their work looked at what students did in ‘equivalent’ by-hand and by-computer items.

The study investigated the effects, at KS2 and KS3, of assessing key stage items on paper and on computer to samples of students in years 6 and 9, in order to study the effects of medium on the material and consequential effects on the performance of pupils. An attempt was made to replicate paper-based questions on the computer. 24 questions, aimed at middle attaining students were assembled into four tests of 12 questions, with each question appearing in two
tests. Each student sat a paper-based test with its complementary computer test. Overall the performance was comparable in each medium: at KS2 students scored 3% better overall on computer than on paper; at KS3 the pupils scored 5% better overall on paper than on computer. Threlfall & Pool (ibid.) examine questions with significantly large differences in facility in one medium or another as shown in the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Paper facility (%)</th>
<th>Computer facility (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KS2 Higher facility on Computer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>39.5</td>
<td>52.7</td>
</tr>
<tr>
<td>Circles</td>
<td>64.5</td>
<td>88.1</td>
</tr>
<tr>
<td>Sum</td>
<td>77.1</td>
<td>94.9</td>
</tr>
<tr>
<td>Diagonals</td>
<td>35.9</td>
<td>67.3</td>
</tr>
<tr>
<td>KS2 Higher on Paper</td>
<td>Blocks</td>
<td>69.0</td>
</tr>
<tr>
<td>KS3 Higher on Computer</td>
<td>Calculation</td>
<td>21.2</td>
</tr>
<tr>
<td>KS3 Higher on Paper</td>
<td>Shapes</td>
<td>49.0</td>
</tr>
</tbody>
</table>

The following focuses on Circles and Shapes to illustrate issues that arise in using a computer in assessing mathematics. Circles states “Here is a grid with eight circles on it”. The paper-based item states “Draw two more circles to make a symmetrical pattern” and the computer-based item states “Move the two extra circles on to the grid to make a symmetrical pattern”. The third part of Shapes is shown. The paper-based item states “Draw a triangle that has an area of $9\text{cm}^2$” and the computer-based item states “Finally move the red dots to make a triangle that has an area of $9\text{cm}^2$.”
Threlfall & Pool (ibid.) comment with regard to Circles:

... two circles have to be located so as to make the overall design symmetrical. On paper, the circles cannot actually be drawn until after a decision has been made about where they should go, because of the mess resulting from a change of mind. The pupil needs to decide that it will look right without being able to try it properly, so has either to be able to visualise, or be analytic – matching pairs across possible lines of symmetry. On computer, the pupil can put the two circles on and make a judgement by recognition – does this arrangement look symmetrical? If not, he or she can move them elsewhere (or, if he or she cannot remember which circles were placed and which were already there, can “start again”). Here the affordance of the computer medium enables easier success – by recognition of symmetry rather than through visualisation or by analysis. If students are willing to try things out, the question is assessing whether they recognise symmetry when they see it, but also should incorporate elements of visualisation and / or analysis. If that is accepted, then the activity afforded by the computer is not legitimate for the assessment, and the computer question is an inferior assessment item.

With regard to Shapes Threlfall & Pool (ibid.) comment:

Prima facie, it might be thought that the computer version would be easier for pupils than the paper version because of the computer affordance of exploratory action – students can try out different shapes ‘for size’ and by that means arrive at a correct solution on a trial and error basis. However, in practice the computer performance is considerably worse than on paper. ... It seems that the computer affordance to enable exploratory action was not as useful as might be supposed. ... On the paper and pencil question there were two observed elements to the typical approach to answering this question. The first was that most pupils began by drawing a horizontal line, and then building a triangle up from it. The second was that many pupils evaluated size by counting squares. ... The computer version of Shapes seems to require a more analytic and strategic approach to the problem than the paper version does. On paper, a piecemeal approach is a feasible attunement to the affordances of the medium, starting with a plausible line, then seeing what it leads to. In the absence of a similar attunement to the computer affordances - for example to begin by extending the horizontal base - pupils probably had to consider the problem in terms of the formula for the area of a triangle, and understand what was needed before changing the shape.

The upshot of this work by Threlfall & Pool (ibid.) is that some ostensibly equivalent items set in paper-based and computer-based media have affordances and constraints which significantly impact on student performance in ways that are difficult to judge a priori. This has obvious ramifications for e-assessment but also for the integration of ICT into assessment considered in this report: some assessment items for which students may use ICT are likely to assist or hinder student solutions in ways that are difficult to foresee in advance. This adds weight to ACME’s (2005) claim that “More research is needed on the use of computer-based assessment in 14–19 mathematics”. Further support for this claim is provided by the report on graphic calculator by Brown & Davies (2002) consider above, as they noted unexpected solution formats in students’ work.

Possible assessment formats
Notwithstanding the problems noted above the discussion moves to consider possible assessment formats for which ICT may be used. Two general forms are considered: coursework (or portfolio work as it is sometimes called) and examinations.

Coursework, in the true meaning of the term, as ‘work done during the course’ is a natural candidate for integrating ICT into assessment. Students can do relevant ICT work at a time when relevant concepts are being studied. Consider, for example, Kenneth Ruthven’s "vignette
1 on conjecturing and proving in geometry using dynamic geometry software described in Appendix 2. A coursework task could be:

Given a circle centred at O, and a chord AB on that circle, Q could be defined:

(i) as the intersection of the line through A perpendicular to AB, with the line through B passing through O; yielding the consequential property that Q lies on the circle.

(ii) as the additional intersection with the circle of the line through A perpendicular to AB.

(iii) as the additional intersection with the circle of the line through B passing through O.

Formulate and prove consequential properties arising from each of definitions (ii) and (iii).

This could be set in the middle of a sequence of lessons on circle work with the curriculum advantage of informing the development of subsequent circle work. An advantage to the students, over placing such work in an examination, is that they have time to explore the properties without the panic that might set in under timed conditions. A logistic advantage to schools is that such work need not make demands on a large number of computers at specific dates/times.

However, serious disquiet about coursework, especially GCSE coursework, amongst a large (apparently a majority) section of the mathematical community means that it is unwise to suggest further coursework, at least for GCSE and probably for GCE too. This matter is returned to in the next section of this report, however, where portfolio work for FSMQ and specialised diplomas are considered.

With regard to examinations this report dismisses pure e-assessment for reasons given by ACME (2005), the danger that such assessment will focus on short structured questions at the expense of higher level skills. There are two potentially significant problems with the integration of ICT into examinations that need to be considered. The first may be called the school/college ‘infrastructure problem’ of requiring, say, 250 working computers at a specific date/time. In the immediate future this appears to prevent the incorporation of desktop computers into assessment unless the logistics of assessment-when-ready suggested that the demand on computers at any given time was manageable. A solution is to ensure there are handheld devices. These could be handheld computers, which would be very expensive for schools, or graphic calculators. An issue that would arise would be to ensure that such devices
were not pre-loaded with software or information that would jeopardise the validity of the examination or give some students an unfair advantage over others. A possibility which would have initial cost implications for the government would be to commission the design and subsequent manufacture of sufficient handheld machines for all 14-19 students.

The second potentially significant problem is that students may need ‘time to play’ with ICT to orientate themselves to the context provided by the assessment questions. This problem is less likely to occur in assessment items where ICT is a tool to be used on occasions in an examination such as the use of graphic calculator in IBO examinations. In all the Strand 1 vignettes and in the assessment questions in Appendix 2 of this report, however, this does appear as a problem. One potential solution, in the absence of suggesting coursework, is to make use of prior data sheets as described in the review of Strand 2. The use of prior data sheets still requires an examination, so plagiarism is not an issue, but does allow students to explore a context or application prior to assessment, arguably alleviating ‘panic’ that could result under the pressurised conditions of an examinations. As mentioned in the section Strand 2, the first two assessment items in Appendix 3 present examples of how prior data sheets may be used in assessment.

**Other assessment issues**

This section considers four independent issues: what is being assessed; the possibility of chaotic grades; greater choice in assessment; and technology-free papers.

The issue of what is being assessed is important to make explicit. For example, in a Strand 2 paper the following question was asked, “In what ways could/should we change L2 qualifications (GCSE) to assess ICT skills in mathematics more explicitly?” It appeared that it was not until this question was asked, that different viewpoints surfaced. The ‘pole positions’ on this matter appear to be: ICT is a now a part of mathematics and there are ICT skills which should be assessed within the assessment of mathematics; although ICT is used in mathematics the situation is no different to, say, the use of a compass, students use the tool but the focus of assessment is students doing mathematics. This, perhaps, is just a value judgement, but an important value judgement to debate.

The issue of ‘possible chaotic grades’ relates to fine gradings of examinations through examiners’ experience. Pencil and paper tests, especially at GCSE and GCE, have a long history of development and refinement. Examiners have a good feel for how students will do and what a C grade and what an A grade student will do on a question. When examiners get this wrong then serious consequences follow. The fact is that there is no tradition of examining high stakes examinations in England and without this tradition gradings could be very unstable. Such instability could have dire consequences, e.g. large numbers of students obtaining unexpected grades. Without extensive experience of assessing mathematics with ICT in England it is difficult to suggest what to do about this potential problem. A possible short-term solution is for mathematics-with-ICT questions to be pass/fail rather than graded.

Choice, in questions which may be answered, was more common in high stakes examinations 30 years ago than it is today. The integration of ICT into assessment raises the question of whether greater choice should be re-introduced. This issue appears more important if assessment expects students to use computer software than assessment requiring graphic calculators. With regard to computer software it appears unlikely that students’ classrooms experiences will, at least in the immediate future, be uniform and will depend on what their teachers can do. Given this students should arguably be given a range of tasks, at least of
which is such that their classroom experiences provides adequate preparation. This argument may not apply to graphic calculators as tools which can be used for a multitude of purposes.

Technology-free papers, papers for which no digital aids are allowed, should be reconsidered in any move to integrate ICT into assessment. Apart from a focus on skills students should exhibit with and without the use of technology, technology-free papers could assuage potential future divisions, as exhibited in the calculator debate, in the mathematics education community. It appears that calculator-free papers were introduced because of concerns of mathematicians who were sceptical about their use in teaching, learning and assessment but technology-free papers could be positively embraced by techno-mathematics enthusiasts along the lines of “OK, let the by-hand skills be examined separately so that students are free to use technology freely in the remaining papers”. Calculator restrictions already exist in Key Stage examinations and graphic calculator restrictions exist at GCE. There is an argument that these should be completely rethought if ICT is to be further integrated into assessment. With regard to GCE a possible format would be an initial technology-free paper in which all by-hand only core skills were examined. Such a paper is currently being considered by the International Baccalaureate Organisation.
The focus of this report up to this point has been with GCSE and GCE. This is because GCSE is the course followed by the largest number of students and GCE is the academic ‘gold standard’. The future of mathematics in school and colleges in England will have other important mathematics courses and qualifications. In this section ICT is considered with regard to functional mathematics, free standing mathematical qualifications and specialist diplomas.

**Functional mathematics**

Functional mathematics does not, at the time of writing, exist except as blueprints in the two Phase 1 reports and in draft Standards. This presents a rather obvious problem with regard to reporting how ICT may be integrated in the curriculum and assessment of functional mathematics. The matter is, however, too important to omit from consideration. In the absence of an agreed description of functional mathematics this report draws on the current, at the time of writing, form of the draft Standards. This has limitations because, as the draft Standards point out “Standards do not ‘stand alone’ for the purposes of teaching, learning and assessment.” The draft Standards (August 2006) for Level 2 are included as Appendix 3; the draft Standards for Level 1 are very similar and use National Curriculum Mathematics levels 1-4.

The three unifying themes are representing, analysing and interpreting. These themes are compatible with a process curriculum, and the scope to integrate ICT into such a curriculum, as discussed in the section *Curriculum matters* above. Further to this many of the bulleted statements are clearly such that ICT use could enhance student activity, e.g.:

- making an initial model of a situation using suitable forms of representation
- deciding on the methods, operations and tools, including ICT, to use in a situation
- changing values and assumptions or adjusting relationships to see the effects on answers in the model
- choosing appropriate language and forms of presentation to communicate results and conclusions

Kenneth Ruthven’s *vignette* 2, see Appendix 2, on representations using spreadsheets and his example activity on population density fits well with these statements and the ethos of the draft Standards. Indeed, the draft Standards read like they were written with spreadsheet use in mind. Although “changing values and assumptions or adjusting relationships to see the effects on answers in the model” is generic to ICT use it has obvious application in ‘tweaking variables’ by changing the value in a spreadsheet cell and noting the effect on linked cells.

The form(s) of assessment for functional mathematics is not yet decided but one could imagine the question on the following, taken from the February 2004 AQA Foundation level Free Standing Mathematics Qualification on *Managing Money*, as an assessment item for level 1 functional mathematics. The question could be amended so that it was done on a spreadsheet.

There appears to be considerable scope to integrate ICT use into the teaching, learning and assessment of functional mathematics. This integration will arguably be easier to enact in a new and innovative course/qualification. The intention to assess functional mathematics ‘when ready’ could reduce the ‘infrastructure problem’ noted in the previous section.
Assuming that these arguments for the integration of ICT into functional mathematics resonate with others, questions remain:

♦ Although spreadsheets have obvious application in the teaching, learning and assessment of functional mathematics, should they be the exclusive focus? There are arguments for and against an exclusive focus on spreadsheets. Arguments for include their importance in the world of work, the wide extent of their potential application (numeric and graphical representation; data handling capabilities) and the advantages of students learning one common ICT tool. The main argument against is the richness of other ICT tools: there are better functional graphic software packages and students should arguably use specialist statistical packages for data handling work.

♦ Although ICT does have obvious potential use in the assessment of functional mathematics it is far from clear that all assessment should be based around ICT use – the ability to “read and understand mathematical information” (draft Standards, Level 2) does not suggest ICT use. So how should the use of ICT be distributed in the assessment of functional mathematics? Should there be ICT and ICT-free papers?

<table>
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<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Cost in farm shop</td>
<td>Cost in supermarket</td>
<td>Saving in farm shop</td>
<td>Saving as a percentage of supermarket price</td>
</tr>
<tr>
<td>2</td>
<td>Carrots</td>
<td>£1.04</td>
<td>£1.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Potatoes</td>
<td>£0.84</td>
<td>£1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Swedes</td>
<td>£0.88</td>
<td>£1.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Leg of lamb</td>
<td>£5.50</td>
<td></td>
<td></td>
<td>£8.49</td>
</tr>
</tbody>
</table>

*Source: Express and Star, Wolverhampton 23 January 2002*

(a) Complete the spreadsheet to show the savings on these chosen items and the percentage savings by buying these items from the farm shop.

Give the percentage saving to the nearest whole number. *(4 marks)*

*Space for working*

(b) State a formula which gives the content of cell D4.

*Answer* *(1 mark)*

(c) On which item is the greatest percentage saving made when buying one of these items from a farm shop?

*Answer* *(1 mark)*
Free-standing Mathematics Qualifications

Free-standing Mathematics Qualifications (FSMQ) were designed to provide an alternative qualification to GCSE and GCE AS Maths for post-16 students who may otherwise not study mathematics beyond compulsory schooling. They were designed with the intention that students could use the mathematics they learnt in their academic or vocational studies.

A wide variety of FSMQ exist at levels 1, 2 and 3. The standard form of assessment is a combination of portfolio work and an examination for which a prior data sheet is provided (the exception is an OCR FSMQ Additional Mathematics which is assessed by examination without a prior data sheet). The uptake of FSMQ is not high and varies greatly over modules but this is probably due to their newness; there does not appear to be any report on them that has not been largely positive.

Geoff Wake, in a report for Strand 2, summarised the demands on the use of ICT as:

♦ use of calculators throughout
♦ use of graphic calculators at advanced level
♦ use of spreadsheets at all levels
♦ use of graph plotting software promoted
♦ use of dynamic geometry and other drawing software in shape and space units (portfolio production only)

There appear to be a number of factors that make FSMQ particularly open to the integration of ICT. These include:

♦ The specialist nature of the modules often makes software choices obvious, e.g. spreadsheets for Managing Money, statistical packages for data handling modules, dynamic geometry for shape and space modules.
♦ The lack of public concern about FSMQ portfolio work. As noted in the section Assessment matters, coursework is a potentially useful format for the assessment of ICT but public disquiet about coursework, especially at GCSE, suggests that it is unwise to recommend that ICT be assessed through coursework. This argument does not appear to apply to FSMQ.
♦ The use of prior data sheets provides a means for students to familiarise themselves with the contexts examinations set questions on – potentially assisting students need to orientate themselves to the context provided by the assessment questions.
♦ The small number of students taking FSMQ in any one school or college means that what is termed the ‘infrastructure problem’ above, effectively disappears.

Consideration of FSMQ are further explored in the section Possible ways forward because the above points appear to make FSMQ a suitable focus on which to base immediate trialling and piloting.

Specialist diplomas

Specialised Diplomas are a proposed qualification for 14 to 19 year old students which are intended to combine practical skill development with theoretical and technical understanding and knowledge. They will be offered at levels 1, 2 and 3 and will have three components:

♦ principal learning – knowledge, understanding, skills and attitudes relevant to a sector
♦ additional/specialist learning – choice from a range of options endorsed by employers
♦ generic learning – skills and knowledge necessary for learning, employment and personal development.
There will be 14 specialised Diplomas:

<table>
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<tr>
<th>Information and communication technology</th>
<th>Health and social care</th>
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<tr>
<td>Engineering</td>
<td>Creative and media</td>
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<tr>
<td>Construction and built environment</td>
<td>Land-based and environmental</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Hair and beauty</td>
</tr>
<tr>
<td>Business administration and finance</td>
<td>Hospitality and catering</td>
</tr>
<tr>
<td>Public services</td>
<td>Sport and leisure</td>
</tr>
<tr>
<td>Retail</td>
<td>Travel and tourism</td>
</tr>
</tbody>
</table>

Functional mathematics, at the appropriate level, will form the mathematical base for these Diplomas but it is recognised that some specialised Diplomas, e.g. Engineering, will require additional mathematics. It is evident from existing documentation that consideration of what this additional mathematics will be has not progressed very far. For example, the draft Engineering level 3 Diploma has a unit *Technology Maths and Science* which comprises 180 guided learning hours, one third of the hours for the principal learning of the Diploma. There are two learning outcomes, for mathematics and for science. The mathematical learning outcome states, “Apply mathematical principles to technology ([Appropriate Maths content taken from GCE Maths and/or National Certificate](#))”. The underlined words suggest both a lack of coherent thinking about mathematics content and that it will taken from elsewhere. Perhaps sustained thinking on this matter may suggest that something akin to an FSMQ may be more appropriate.

As with, and for similar reasons to, functional mathematics and FSMQ, the integration of ICT into the teaching, learning and assessment of the mathematics component of specialist Diplomas appears highly desirable and potentially less problematic than the integration of ICT at GCSE. Specifying more at this stage would be over speculative.
8 Curriculum and assessment principles

Many principles can be penned. The NCTM, for example, currently has six principles for school mathematics. One regards technology, it states:

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning. ([http://www.nctm.org/standards/#annuals](http://www.nctm.org/standards/#annuals))

This principle expresses a value judgement regarding the role of technology in mathematics teaching and learning. Many mathematics educators in England would dispute the claim that technology \textit{per se} “enhances students’ learning”. This principle does not suggest means for the integration of technology into mathematics teaching, learning, curricula or assessment. In contrast to this NCTM principle, the principles put forward below attempt to focus on concrete issues which may inform curriculum and assessment design.

Curriculum and assessment principles for the integration of ICT into the mathematics curriculum and assessment are useful as a means to make the implicit aims, of parties involved in the debate, explicit. The following principles are offered as principles to be discussed. Comment on each principle follows each statement.

Curriculum principles

1) No topic, concept, skill or process currently addressed in mathematics curricula should be excluded for mathematical work using ICT.

Comment The intention behind this principle is to avoid ICT being viewed as applicable to specific topics, concepts, skills or processes. Whilst certain topics, concepts, skills or processes may be deemed more or less suitable for mathematical work using ICT, this represents what is known/believed at this moment in time. The use of ICT in mathematics, relative to the history of mathematics, is in its infancy and it is to be expected that there is much that can be done for which nothing is known at the moment; to prescribe the application of ICT in mathematics could stifle future desirable application. This principle does not fit well with the suggestion, made at the end of the section on Assessment matters, of technology-free papers. So be it; individuals may wish to delete or qualify the term ‘skill’.

2) The use of ICT should enhance the curriculum by viewing existing mathematics in new ways.

Comment Tool use transforms mathematical action. This has been described several times in this report, e.g. the third paragraph in the section Curriculum matters. Given that mathematics with ICT will be different than mathematics without ICT it is clearly desirable that it should enhance students’ experience of doing mathematics. The Strand 1 vignettes provide details of how ICT can be used to enhance students’ experiences of doing mathematics.

3) When new mathematical foci appear as a result of integrating ICT into curricula, then a suitable timeline should be provided prior to assessment to ensure that students are adequately prepared.

Comment The first two principles concerned existing mathematics but ICT brings new mathematics and new ways to view existing mathematics, hence a principle on ‘things to come’. This principle is intended to ensure that teachers and students are adequately prepared for changes in the curriculum. For example, if ‘representation and precision of spreadsheet numeric values’ (see Kenneth Ruthven’s vignette 2 in Appendix 2) is integrated into the curriculum and assessment, then teachers need to know this well in advance to be able to...
prepare their students. This principle may be stating the obvious but it seems wise to err on the side of clarity.

Assessment principles
1) Assessment should be of students’ mathematics, not of students’ ICT skills.
Comment This is a contentious principle. As the report on the Strand 2 work shows, there are mathematics educators who disagree with it. Those who support this principle view ICT as a tool in mathematics (a powerful tool but still a tool). When, they say, a student is assessed on the use of a compass or protractor in mathematics, it is the mathematics that is assessed, not the tool use (even though the tool must be used properly). Against this view there is distinct body of opinion that some ICT skills, e.g. formatting cell precision in a spreadsheet, are mathematical skills which can/should themselves be assessed. This issue merits wide debate.

2) Assessment items should not specify ICT applications.
Comment There are two levels to which this principle may apply. One is the level of not specifying brand names, e.g. Cabri or Geometer’s Sketchpad. It is likely that few would disagree with this. Another level is not specifying any platform. An example may clarify the issues at hand. The Transport costs draft assessment item in Appendix 3 does not specify a platform in the prior data sheet (an oversight on the part of the author) but the assessment clearly expects a spreadsheet solution, directing students to go “from 1 to 20 in steps of 1 gallon”. This was criticised for being too ‘leading’. But the substance of this set of tasks does not require tables or spreadsheets, it can be done with a graph plotter. So should a spreadsheet be specified at all? This principle says no. Further to this Noss & Hoyles (1996) give an account of by-hand, Cabri and Logo solutions to a problem. The Logo solutions surprised other parties by its novelty. This principle is clearly contentious but is intended to avoid ruling out students’ novel solutions and/or solutions using ICT tools which they have appropriated.

3) Assessment items must be written to ensure a complete range of student success.
Comment Those who have experienced writing trial assessment items which may be done using ICT often report that writing conceptually challenging items is easier than writing items of a more pedestrian nature. More worrying than this they often report, see Monaghan (2000), that expected ICT use encourages them to omit easier parts of equivalent by-hand items. This is surely undesirable. On the other hand it is important that attempts at equity for lower attaining students do not prevent examiners writing challenging items. There is a future significant challenge for examiners in realising this principle.
9 Possible ways forward

This report is written: as a discussion document; as a means of collecting together various thoughts on ICT in the mathematics curriculum and assessment; to address the region between Strand 2 and Strand 3; and as a source of ideas that might feed into Phase 2 work. This final section draws together matters that might take work further. There are three subsections. The first summarises issues arising from sections 2 to 7. The second makes tentative recommendations for the inclusion of ICT into specific qualifications. The final outlines possible trialling and piloting that could commence in the immediate future.

Summary of issues arising from sections 2 to 7

Section 2 Strand 2 work raised a number of issues:
♦ Value was seen in a wide range of software and microworlds but three generic software systems were seen as particularly relevant: spreadsheets, dynamic geometry and function graphing packages. Computer algebra systems and statistical packages were viewed positively but present specific problems regarding their introduction at this time. There was disagreement on the privileging of the place of spreadsheets in the curriculum.
♦ Regarding hardware: schools do not currently have sufficient desktop computers to ensure that computers can be integrated into compulsory examinations; laptop computers offer a way to alleviate the desktop problem and offer portability but a concern is with costs; graphic calculators can overcome the above problems but their functionality, compared to computers, are currently limited; pre-loaded programs may give some students an unfair advantage
♦ Given the current ethos of ‘teaching for the test’ curriculum changes should be matched by assessment changes. ICT offers a means to assess-when-ready. There was disagreement as to whether ICT skills should be assessed within mathematics.
♦ Forms of assessment presented various opportunities and constraints: coursework was viewed as a suitable means to assess the use of ICT in mathematics but current concerns make this a problematic vehicle for assessment at GCSE; traditional examinations were seen as extremely problematic for platforms other than graphic calculators; the use of prior data sheets was something worthy of further consideration.

Section 3 The English experience, to date, of using graphic calculators presents problems on which to develop future integration of graphic calculators into curriculum and assessment. There is much that can be learnt from the International Baccalaureate Organisation which has adopted a progressive and considered approach to the use of graphic calculators. Even so problems arise: developing examination questions; the preparedness of examiners and of teachers; mark schemes and (in)appropriate student responses; the variety of graphic calculators and their functionalities; syntax and display problems; a long ‘lead in’ time to prepare schools.

Section 4 It is important to consider target groups. Separate consideration of the needs of adults and of Entry level students is required. With regard to levels 1, 2 and 3 there is sense in considering, and perhaps even prioritising, ways in which ICT can be integrated into the teaching, learning and assessment of level 1 mathematics for otherwise their technomathematical experiences could be impoverished.

Section 5 There are problems with integrating ICT into a content curriculum. A process curriculum appears to present fewer problems for the integration of ICT (though other problems may arise). The four key processes (representing, analysing, interpreting and
appreciating) of the draft KS4 Programmes of Study appear to be suitable processes on which to base the integration of ICT into the curriculum. It is important to consider ICT opportunities in the enacted classroom curriculum. Statistics is an area of mathematics for which the integration of ICT is particularly suitable but the current review of the place of statistics in GCSE Mathematics, and ramifications for statistics in other courses, presents problems for developing this area at the moment.

Section 6 The integration of ICT into the assessment of mathematics is likely to be a controversial matter with members of the mathematics education community adopting various positions as to its worth. Little is known about assessing mathematics on a computer and research suggests that some ostensibly equivalent items set in paper-based and computer-based media have affordances and constraints which significantly impact on student performance in ways that are difficult to judge a priori. More research is needed on the use of ICT in the assessment of 14-19 mathematics. Regarding assessment formats: coursework is suitable but presents problems at GCSE; e-assessment presents the danger that assessment will focus on short structured questions at the expense of higher level skills; traditional examinations present an ‘infrastructure problem’ for schools/colleges and students may need time to orientate themselves towards the contexts of examination questions. The use of prior data sheets in examinations requiring the use of ICT appears to present a solution to the ‘time for students to orientate themselves’ problem. Other assessment issues are: what is being assessed; the possibility of chaotic grades; the possible need for greater choice in assessment; and technology-free papers.

Section 7 Courses other than GCSE and GCE considered were functional mathematics, free-standing mathematics qualifications and specialised Diplomas. A problem with functional mathematics and specialised Diplomas is that their development is ongoing, which limits what can be said regarding the integration of ICT. All of these courses were seen as having considerable scope for the integration of ICT into their curriculum and into their assessment. Free-standing mathematics qualifications were viewed as particularly suitable for the integration of ICT.

Tentative recommendations for the inclusion of ICT into specific qualifications
This subsection is included at the request of QCA staff. It is understandable that QCA wishes to consider specific forms of ICT use for different course as ‘anything goes’ presents serious problems for progressing from the current position. But a problem in recommending a specific ICT application, e.g. spreadsheets, for a particular course, e.g. GCSE, is that this specific ICT form is then privileged over others. With this caveat the following recommendations are offered for discussion.

NB The following names specific ICT systems, e.g. spreadsheets, but it should be recognised that the most appropriate uses of these tools may be to create microworlds within these systems along with specifically designed educational tasks.

**GCSE Scientific calculators, spreadsheets, dynamic geometry**
♦ Greater emphasis on number formats with scientific calculators.
♦ Using spreadsheets to represent numbers in different ways, structure data in suitable ways, ‘tweak’ variables and manage relationships.
♦ Using dynamic geometry to make conjectures and support the process of proving.
Although a wide variety of software may be used in teaching and learning, it is feasible to advance the integration of ICT into GCE curriculum and assessment through integrating graphic calculators with the following functionalities into teaching, learning, curriculum and assessment: Cartesian, parametric and polar graphs; built-in numeric routines and functions; programming; tables; advanced statistical features. NB Functionalities used will vary over modules.

The emphasis of functional mathematics is expected to be practical with a focus on modelling, representing, analysing and interpreting. It would appear sensible that tools, for specific significant tasks, are not simply presented but that students are involved in deciding on the appropriate tool. ‘Statistical packages’ is bracketed simply due to the current review of the place of statistics in GCSE Mathematics, and possible ramifications of this for functional mathematics.

A wide variety of software depending on the specific nature or the FSMQ unit or specialised Diploma.

What is clear from the above considerations is how little is known and how little has been done to date. The work of Brown & Davies (2002) and of Threlfall & Pool (undated) shows how surprising and unexpected assessment using ICT can be; there is much that will not be known until experiments are made. This reinforces ACME’s (2005) call for more research. Research can be conducted for many purposes and in many ways. There is a sort of Catch 22 with regard to research which will inform future curriculum and assessment development with regard to ICT: research which does not address real teaching, learning and, crucially, assessment is unlikely to capture the efforts (from teachers and students) that go into getting good grades in real exams; on the other hand experimenting with students’ education is rife with serious ethical issues when so little is known at the outset. A way out of this dilemma is to develop curricula and assessment in an area where the use of ICT in teaching, learning and assessment appears most likely to enhance learning and lead to equitable grading in assessment. The discussion in section 7 suggests that FSMQ present such an area. This is presented as a recommendation.

Phase 2 contractors add to their trialling and piloting, work on three new or (amendments of) existing FSMQ, one at each of levels 1, 2 and 3, which would involve the systematic integration of ICT into curriculum, teaching, learning and assessment. The contractors to keep detailed records of all developments, outputs, teacher development and student work and to report on these at appropriate times. Independent research be commissioned to evaluate this work.

This work does not prevent parallel developments being carried out in other areas. The upshot of this work can reasonably be expected to provide a knowledge base for future developments.
References


Threlfall, J. & Pool, P. (undated) ‘Implicit aspects of paper and pencil mathematics assessment that come to light through the use of the computer’. Working paper currently being prepared for submission. Available on request from J.Threlfall@education.leeds.ac.uk
Appendix 1  Draft document from QCA on ICT in Mathematics

ICT in Mathematics – outline of phase 2 ‘qualification’ deliverable

Objective: To identify a set of principles about qualification design and criteria that will enable ICT to be used more effectively in mathematics.

Rationale: The curriculum at 14-19 is generally determined by the specification of qualifications. It therefore makes sense to concentrate initial efforts on deciding how new qualifications should reflect and promote the links between mathematics and ICT. This will inform the development of new mathematics qualifications and feed into the work on curriculum pathways arising from the Smith Inquiry and the 14-19 education and skills white paper.

Questions to address
What should a qualification specification in mathematics look like, in particular

a. What should it say about ‘cognitive skills’, in mathematics or in ICT, as opposed to techniques?
b. How should it identify opportunities to link mathematics and ICT?
c. How should it avoid being an impediment to the integration of mathematics and ICT?
d. What forms of assessment should it propose?
e. Would it need to specify teaching approaches as well as topics?

Outline of work
There are four Strands of work as follows:

Strand 1: Existing qualifications –
Undertake a survey of a broad range of current qualifications in mathematics with the aims of identifying elements of helpful practice in relation to ICT, and on the other hand seeing what things might be obstacles. This survey should not be confined to the specifications only but should include all relevant aspects of the qualification, in particular the assessment instruments.

Strand 2: Tentative answers –
While working on Strand 1, attempt to provide answers to the questions a. – e. above. It is likely that Strand 1 will suggest not only answers, but also further questions to take into consideration.

Strand 3: Redesign an existing qualification –
Take one of the qualifications surveyed in Strand 1 and attempt to rewrite the specification in order to take account of the answers provided in Strand 2. This might be extended to the assessment instruments too.

Strand 4: Evaluation –
Consider to what extent the work so far has provided sound principles on which to base qualification design in the medium term. Decide whether the conclusions are sufficiently firm to be shared with awarding bodies to inform design of mathematics qualifications. Identify what specific further work is necessary.
Appendix 2  *Strand 1 vignettes*

1) Federica Olivero and Rosamund Sutherland of the Graduate School of Education, University of Bristol provided two vignettes based around the use of dynamic geometry.

**Vignette 1  Conjecturing and proving in geometry using dynamic geometry software**

The aim is to show how dynamic geometry can transform and support the process of conjecturing and proving. Specific aims with regard to student work are:

- to read the mathematical relationships emerging from their exploration
- to notice relevant properties through the process of construction
- to pose questions and make conjectures
- to validate conjectures
- to reorganise discovered properties into logical deductions
- to prove validated conjectures.

Varignon’s problem is used to illustrate how these aims may be realised:

**VARIGNON’S PROBLEM**

![VARIGNON’S PROBLEM Diagram](image)

- Draw any quadrilateral ABCD. Draw the midpoints L, M, N, P of the four sides.
- Investigate the properties of the quadrilateral LMNP.
- Which particular quadrilateral can LMNP become?
- Investigate how LMNP changes in relation to ABCD.
- Prove your conjectures.

It is important to note, as the authors do, that “dynamic geometry can transform and support the process of conjecturing and proving”. Dynamic geometry, like any tool, transforms mathematical actions, i.e. they are not the same actions as ‘by-hand’ actions. Dynamic geometry does not prove (or otherwise) conjectures – people prove – but they may support proofs.

With regard to proof dynamic geometry can provide tremendous support to students in *seeing*, by dragging vertices, mathematical invariants (parallel sides in the case of this problem). It is crucial that students do not regard the dragging process as a proof.

The vignette suggests activities and provides examples of pupils working on this and related problems. The vignette ends with curriculum considerations and notes that:
This vignette addresses in particular the following objectives:
- KS4 foundation/ Shape, space and measures 1f, 2f, 4e;
- KS4 higher/ 1a, 1f, 1g, 1h, 1i, 1j, 2b, 2c, 2d;
- GCE Advanced & AS/3.2 (a)

... Learning about geometrical properties tends to be separated from justification and proof. The missing link in the current curriculum is between spotting patterns/conjecturing and the justification and proof of these conjectures. ... Dynamic geometry has the potential of supporting students to develop at the same time conjectures, justifications and proof in geometry. The domain change will involve a shift in emphasis towards the relationship between conjectures and justification and proof. This will involve paying explicit attention to geometrical properties and the role of deduction in geometry. ... Work with dynamic geometry will enable students to rapidly develop conjectures about geometrical properties through what they 'see' as they drag geometrical objects. This is different from being ‘told about’ properties by a teacher. ... if dynamic geometry is ever to make a difference about developing an approach to conjecturing and proving that supports the construction of a system of mathematics knowledge, then it needs to be an integral part of the curriculum and used in a systematic way.

**Vignette 2  Transformations and dynamic geometry**

The aim is to show how dynamic geometry environments:
- allow concrete and accessible work with geometrical
- make visible the mathematical relationships that characterise each transformation
- support the construction of a global classification for transformations

Activities which can be carried out include:
- developing the mathematical definition of each transformation through exploration
- constructing transformations according to their mathematical definition
- recognising transformations, through exploring 'black box' activities
- using transformations to solve problems
- recognise transformations performed by specific mathematical mechanisms

With regard to the curriculum the authors note that this vignette addresses:

- KS4 Shape, space and measures foundation/ 3a-b-c-d, KS4 Higher/ 3c.

The ‘big idea’ here is to introduce students to mathematical transformations as a systematic organised body of knowledge.

This differs form the current focus which studies each transformation separately.

Essential change to the current curriculum: Similarities are not currently included in the curriculum as transformations but only similar figures are introduced. Introducing similarities as part of transformations comes as a extension of the work on isometries and takes the same form of activity in a dynamic geometry environment.
2) Kenneth Ruthven, School of Education, University of Cambridge provided three vignettes.

**Vignette 1** Conjecturing and proving in geometry using dynamic geometry software

This vignette looks at how dynamic geometry can support geometric reasoning processes: conjecturing, testing and proving. This is illustrated by means of the example below.

A chord $AB$ is constructed and lines, through $A$ perpendicular to the chord and through $B$ through $O$, are constructed – why is the intersection of these lines always on the circle? Adding a line segment $OA$, marking angles and dragging $A$ (no effect on $Q$) and $B$ can support students’ conjectures and highlight mathematical relationships.

The vignette goes on to consider current practice and prospective practice. Current practice uses “numeric pattern-spotting which has become widely used in English schools ... effective preparation for the kinds of ‘angle-chasing’ exercises which predominate in current texts and tests.” Prospective practice can design activities “to bring out and develop the system-building, visuo-spatial and logico-deductive aspects of geometrical reasoning.”

With regard to the curriculum the author concludes:

Recent revisions to the current NC at Higher level have sought to introduce greater emphasis on mathematical reasoning in geometry. Given a serious commitment ... to a more sustained and substantial development of geometrical reasoning in Higher level GCSE, we recommend adopting dynamic geometry software as a standard tool capable of making an important contribution to such a development.

**Vignette 2** Representation and precision of spreadsheet numeric values

This vignette examines how issues of the presentation and precision of numeric values arise in the course of using a spreadsheet. Key mathematical ideas centre around the distinction between the values entered-stored-displayed, the number formats employed and the tools provided to manage representation and precision of numeric values. The author notes that:

Awareness of such factors, and facility in recognising, interpreting and managing them, are fundamental to developing understanding and skill in making mathematical use of computational technologies.
An activity is provided on the population density of London local authorities:

<table>
<thead>
<tr>
<th>Region [Local Authority]</th>
<th>Population [people]</th>
<th>Land Area [square miles]</th>
<th>Population density [people/square mile]</th>
</tr>
</thead>
<tbody>
<tr>
<td>London Barnet</td>
<td>331,500</td>
<td>34</td>
<td>9750</td>
</tr>
<tr>
<td>London Bexley</td>
<td>217,800</td>
<td>24</td>
<td>9075</td>
</tr>
<tr>
<td>London Brent</td>
<td>253,200</td>
<td>17</td>
<td>14894.1176</td>
</tr>
<tr>
<td>London Bromley</td>
<td>297,100</td>
<td>59</td>
<td>5035.59322</td>
</tr>
</tbody>
</table>

This is an appropriate activity for students to explore various methods of rounding numbers and different number formats and their representations as well as extension activities such as sorting, index values and lower/upper bounds for the populations, areas and populations densities.

With regard to the curriculum the author notes that current curriculum guidance on the use of calculators should be extended to the use of spreadsheets. This should include entering complex calculations and calculations in a wide range of number formats. Beyond analogies to calculator use spreadsheets introduce new mathematical ideas: repeating calculations across a set of cases and sorting arrays. Further to this:

This distinction between stored and displayed values means that there is an important difference between copying and retyping a cell value. This relates to the current NC requirements that pupils be taught not to round values during the intermediate steps of calculation ... efficient use of a spreadsheet calls for the capacity to recognise and manage a range of relationships between stored values and different forms of displayed value, concerned with issues both of type of format and degree of rounding. ... We suggest that all these media of calculation be acknowledged in the curriculum, but that the weighting accorded to developing proficient written calculation be reduced.

**Vignette 3** Graphic and symbolic analysis of function variation

This vignette looks at graphic and symbolic manipulation in the context of function variation. The rationale for inclusion of this vignette is founded on the increasing use, in mathematics, in other areas of education and in employment, of digital tools which make use of symbolic and, particularly, graphic facilities, and the need for students to make mathematical sense of these outputs.
A fundamental mathematical idea “is that the image of a graph depends not just on the numeric data or symbolic expression taken as defining it, but on the way in which the axes are scaled, and ... on the medium in which the image is graphed.” $y=x$ is perpendicular to $y=-x$ and $y=\sqrt{1-x^2}$ is semi-circular only if the axes are scaled in a particular manner.

These considerations lead to a transformation in the importance of scaling and of range: “if students are to make effective use of graphing technology within school ... such issues require explicit curricular attention”. Technology facilities such as ‘zooming’ (and varieties of zooming) become mathematical foci and lead to new curricula foci such as local linearity.

The mathematical activity presented is the much referred to ‘maxbox’ but the focus is on mathematical interpretation and graphic and symbolic displays. Curriculum considerations make two specific suggestions:

- that the NC and GCE specifications should now require use of graphware in their treatment of functions and graphs, identifying necessary capacities and concepts more explicitly; and encourage coordinated development of use of the wider symbolic facilities of CAS,

- that the curriculum needs to explicitly aim to develop an appropriately balanced use of traditional and digital media for graphing and symbolic reasoning, notably a well-coordinated understanding of their distinctive tools and techniques.
3) Richard Noss, Lulu Healy and Niall Winters, London Knowledge Lab
   Things to do with ICT in the Mathematics Classroom
   Six vignettes using four microworlds

   NB This set of vignettes comprises, for each vignette: plugins, activities and extensions, ideas for the classroom and curriculum considerations. This review merely copies the opening descriptions of the microworlds explored in the vignettes.

**Mathsticks**  Lulu Healy
   Ideas about sequences
   Ideas about proof

   The Mathsticks Microworld provides a context for constructing and exploring structures and properties associated with positive whole numbers. It consists of a tool set through which visual and symbolic representations of numbers and number sequences, in the form of sets of matches or dots, can be generated and manipulated. The Mathsticks Microworld can also be used to introduce students to aspects of proof and especially to making links between actions with particular cases and the formal language by which these actions can be generalised.

**Chancemaker**  Dave Pratt
   Mending broken tools
   Compound events

   The Chancemaker microworld provides a context to explore concepts related to probability, such as randomness, variability and the law of large numbers, in order that learners can develop notions of distribution. It provides learners with a series of gadgets, mini-computational devices that simulate everyday random generators (a coin, a spinner, a dice and so on), whose outputs can be displayed in a variety of different ways (through lists, pictograms and piecharts) and whose workings can be examined and modified so that possible biases in the associated distributions can be identified and eliminated.
Dynagraph  E Paul Goldenberg  Exploring functions

The Dynagraph microworld can be used to introduce students to the ideas of variable and functional dependency. It provides students with dynamic representations of functions (dynagraphs), in which the domain and range are simultaneously displayed on two parallel axes. In dynagraphs, unlike in symbolic expressions or Cartesian function plots, the variation of the function variable can actually be experienced physically and visually -- it can be dragged along its axis. As the function variable is moved by the learner, the function value moves correspondingly along a parallel axis.
**Turtle Transforms**  Lulu Healy  Reflections on reflection

The Turtle Transformation Microworld has been developed for constructing and exploring geometrical transformations. It consists of a tool set which allows users to communicate with multiple turtles in order that they might construct particular relations between them. The tools are presented in such a form as to provide learners with support in constructing formalizations which encapsulate particular transformations, in the form of mappings of one turtle (or turtle set) onto another. The activities in this example focus on the transformation reflection.
Appendix 3  *Strand 2* tasks with assessments that would require the use of ICT

**Aviaries**

**Introduction**

The threat of bird 'flu’ is a cause of great concern to free range poultry keepers. Many of them keep just a few chickens and allow them to live a natural life scratching around in a field.

If the dreaded H5N1 variety of avian influenza takes hold, these chickens will have to be kept out of contact with all wild birds, typically in sheds. This would cause great distress to the birds and their owners.

Sam decides to set up a business making aviaries for free range poultry keepers. She has drawn this artist’s impression of one of her aviaries.

The aviaries are rectangular enclosures with vertical posts every 2 metres, both around the perimeter and also inside. The aviary in Sam’s picture is 8 metres long and 4 metres wide.

The posts are 2.50 metres long; they are dug into the ground to a depth of 50 centimetres so that 2 metres appear above ground. The hole for each post is filled with concrete.

The posts around the perimeter are connected by three horizontal wooden runners, one at ground level, one in the middle at height 1 metre and one at the top. There are also horizontal wooden runners over the top of the aviary; they are parallel to the sides and connect the tops of the posts on the perimeter to those inside.

The perimeter and top are covered with small gauge wire netting that will not allow birds to get through. The netting that Sam uses is 2 metres wide.

Each aviary has one gate. This is 1 metre wide and just 2 metres high and its design is shown in another of Sam’s drawings. It is made out of the same material as the wooden runners. It has 3 hinges, 3 bolts and one handle. The door requires one extra vertical post.
The task NB This would be on a new sheet in an assessment

Sam needs to know how much to charge for an aviary. She must allow for

- materials
- time
- profit.

1. Using the information on the fact sheet, estimate the cost of the $8 \text{ m} \times 4 \text{ m}$ aviary in Sam’s picture. Show all your calculations clearly.

2. Set up a spreadsheet that will do the calculation for different possible sizes of aviary, like $2 \text{ m} \times 2 \text{ m}$, $4 \text{ m} \times 2 \text{ m}$, $6 \text{ m} \times 2 \text{ m}$, … , $20 \text{ m} \times 10 \text{ m}$, and so on.

Print out the costs for 10 different sizes of aviary. Use the information in the fact sheet.

3. Now set up a new spreadsheet that allows you to enter different costs of materials and labour from those in the fact sheet.

Print out a new fact sheet with your own costs of materials and labour. Also print out the new costs for the 10 sizes of aviary you chose in part 2.
Fact sheet

**Materials**
- **Vertical posts**: £15 for 6
- **Concrete**: 50 p per hole
- **Horizontal runners**: 80 p per metre
- **Netting**: £110 for a 50 metre roll
- **Hinges**: 70 p each
- **Bolts**: 80 p each
- **Door handles**: £1-50 each
- **Screws, nails and staples**: 20 p per vertical post allowed

**Time**
- **Vertical post**: 30 minutes each
- **Horizontal runners**: 10 minutes for each crossing with a vertical post
- **Netting**: 5 minutes per square metre
- **Gate**: 2 hours in total
- **Labour**: £8 per hour

**Profit**: 30% of total costs
Transport costs
Prior data sheet

A transport company uses lorries to transport goods by road. The manager wants to recommend an economic speed for drivers. The drivers have a fixed hourly wage of £18 per hour. Diesel fuel costs 85p per litre. The manager has calculated that fuel consumption at an average speed of 45 miles per hour (mph) is 10 miles per gallon (mpg) and that this goes down by 0.1 mpg for each extra mph over 45 mph.

If the lorry goes too fast, then fuel consumption will increase and fuel costs will go up. But if the lorry goes too slow, then the journey will take longer and the driver’s wages will go up. The manager wants to suggest a speed that keeps both fuel costs and drivers’ wages down.

Notes
♦ 1 gallon ≈ 4.546 litres but you can use 1 gallon = 4.5 litres in any calculations.
♦ Fuel (petrol and diesel) is sold in litres but it is still common to refer to mpg when talking about fuel consumption.
♦ The manager’s fuel consumption calculation is not exact but it is a pretty good estimate.

Trial tasks
For a 100 mile journey, make calculations and create tables and graphs for:
♦ converting gallons to litres and litres to gallons,
♦ miles per gallon at various speeds,
♦ driver, fuel and total costs at different average speeds.
Assessment

Level 1
Remember to label all the columns in your tables and all the axes in your graphs.

1) Create a conversion table ‘gallons to litres’ with gallons going from 1 to 20 in steps of 1 gallon.

2) Create a table to show driver costs for a 200 mile journey for average speeds of 45 to 70 mph in steps of 1 mph.

3) Create a table to show miles per gallon for average speeds of 45 to 70 mph in steps of 1 mph.

4) If diesel fuel costs £0.85 per litre and there are 4.5 litres in a gallon, then how much does diesel fuel cost per gallon.

5) Create a table to show fuel costs for a 200 mile journey for average speeds of 45 to 70 mph in steps of 1 mph.

6) Use your results from parts (2) and (5) to create a table to show total costs for a 200 mile journey for average speeds of 45 to 70 mph in steps of 1 mph.

7) a) Suggest an average speed for drivers.
   b) Say why you have suggested this speed.

Assessment

Level 2
Remember to label all the columns in your tables and all the axes in your graphs.

1) Create a conversion table ‘gallons to litres’ with gallons going from 1 to 20 in steps of 1 gallon.

2) This question has three parts and is based on a 200 mile journey. For each part you should construct a table with column headings ‘Speed’ and ‘Cost’ for average speeds of 45 to 70 mph in steps of 1 mph.
   a) Show the driver costs for average speeds from 45 to 70 mph in steps of 1 mph.
   b) Show the fuel costs for average speeds from 45 to 70 mph in steps of 1 mph.
   c) Show the total cost (wages and fuel) for average speeds from 45 to 70 mph in steps of 1 mph.

3) a) Suggest an average speed for drivers.
   b) Say why you have suggested this speed.

4) A world oil shortage has doubled fuel prices. Adjust your tables in question 2 and suggest a new average speed for drivers.
Cutting the cake

Where should you make a single straight cut with a knife to cut a rectangular piece of cake exactly in half?

What if the cake has icing of different thickness on two sides... where should you make a single cut then so that two people get equal amounts of cake and equal amounts of icing?
The Arbelos

The diagram below shows two circles carefully constructed inside another. The second diagram shows part of this - the arbelos - a shape used as an ancient shoemaker's knife.

By using different sizes of inner circles different shaped arbelos can be formed.
The arbelos has some very interesting geometrical properties.

1. Investigate the circumferences of the three circles, and therefore the perimeter of the knife for different shaped arbelos.

This diagram shows an additional circle constructed with its diameter along the common tangent to the two smaller circles and so that it just touches the diameter of the largest circle.

2. Investigate the area of the arbelos and the area of this circle for different shaped arbelos.

3. This additional circle intersects the smaller circles at a single point and at two other points one on the circumference of each circle. The line drawn through these two points is a tangent to each of these smaller circles. Investigate and explain this property.
Triangle in a circle

Perimeter $\triangle ABC = 17.0 \text{ cm}$

Area $\triangle ABC = 11.9 \text{ cm}^2$

What is the largest triangle that you can draw in a circle?
Investigate.
## Appendix 4  Extract from draft Standards for functional Maths (August 2006)³

### Functional Mathematics Standards

#### Level 1

<table>
<thead>
<tr>
<th>Representing</th>
<th>Analysing</th>
<th>Interpreting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making sense of situations and representing them</td>
<td>Processing and analysing the mathematics</td>
<td>Interpreting and communicating the results of the analysis</td>
</tr>
<tr>
<td>- realising that a situation has aspects that can be represented using mathematics</td>
<td>- using appropriate mathematical procedures</td>
<td>- interpreting results and solutions</td>
</tr>
<tr>
<td>- making an initial model of a situation using suitable forms of representation</td>
<td>- examining patterns or relationships involving a single operation</td>
<td>- drawing conclusions in the light of the situation</td>
</tr>
<tr>
<td>- deciding on the methods, operations and tools, including ICT, to use in a situation</td>
<td>- changing values to see the effects on answers</td>
<td>- considering the appropriateness and accuracy of the results and conclusions</td>
</tr>
<tr>
<td>- selecting the mathematical information to use</td>
<td>- finding results and solutions</td>
<td>- using appropriate language and forms of presentation to communicate results and conclusions</td>
</tr>
</tbody>
</table>

³ The content corresponds to Adult Numeracy standards and Application of Number key skill at Level 1

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³ At the time of writing the author was unable to obtain electronic copy of version 1.2 (August 2006) which is referred to in section 7 of this report. This extract is from version 1.1.
Level 1 (continued)

students apply knowledge and skills from National Curriculum Mathematics’ Levels 1–4 to problems involving:

Quantity
- different ways of presenting numbers including fractions, decimals and percentages
- mental and written calculations and use of a calculator
- everyday measures including money and measuring scales

Space and shape
- properties of 2-D and 3-D shapes
- describing location and movement
- estimating and determining lengths, angles, perimeters and areas

Change and relationships
- patterns and sequences
- formulae expressed in words
- relationships between number operations

Uncertainty
- discrete data, including data presented in tables, charts and diagrams
- statistical measures, including mode and range
- estimates of risk and uncertainty

* The content corresponds to Adult Numeracy standards and Application of Number key skill at Level 1